2. IF YOU HAVE TAKEN DIFFERENTIAL CALCULUS BEFORE, DO NOT USE DIFFERENTIATION SHORTCUTS. 3. YOU SHOULD ONLY REQUIRE A CALCULATOR FOR QUESTIONS MARKED [C]. UNLESS A GRAPH IS GIVEN, YOU MUST BE ABLE TO SOLVE EACH PROBLEM WITHOUT A GRAPH. 4. Estimate the slope of the tangent line to the curve $y = \sqrt{x} + \sqrt{\cos x}$ at the point (0, 1) using the slopes of several secant lines. [1][C] The position of an object (in meters) at time t seconds, is given by the function $f(t) = t^2 \cos \pi t$. Find the average velocity of the [2] object over the interval [1, 5]. Specify the units. [3] Sketch the graph of a function f(x) which satisfies the following conditions: $\lim_{x \to -2^+} g(x) = -3, \qquad \lim_{x \to -2^-} g(x) = \infty, \qquad \lim_{x \to 1} g(x) = -\infty, \qquad \lim_{x \to -\infty} g(x) = 2, \text{ and } \qquad \lim_{x \to \infty} g(x) = -2$ Prove that $\lim_{x\to 0} x^4 \cos \frac{1}{r^2} = 0.$ [4] Let $f(x) = \begin{cases} 2x - 3 & \text{if } x < -1 \\ x^2 - 6 & \text{if } -1 < x < 2 \\ 4x - 6 & \text{if } x \ge 2 \end{cases}$ [5] Find $\lim_{x \to -2} f(x)$. Find $\lim_{x \to -1} f(x)$. [a] [b] Find $\lim_{x\to 2} f(x)$. [c] Find the value of *a* if $\lim_{x\to 2} \frac{\sqrt{x^2 + a} - 1}{x-2} = 2$. [6] If $\lim_{x \to 2} f(x) = -3$ and $\lim_{x \to 2} g(x) = 4$, find $\lim_{x \to 2} \frac{x^2 g(x)}{1 + f(x)}$. Show clearly how the limit laws are used in your solution. [7] Find the discontinuities of $f(x) = \frac{x+2}{x^2-9}$, and find the one-sided limits at each discontinuity. [8]

[9] Let
$$f(x) = \begin{cases} 2x + a & \text{if } x < -1 \\ 3 - x & \text{if } -1 < x < 2 \\ bx - 1 & \text{if } x \ge 2 \end{cases}$$

[a] Find the value of a so that f(x) is continuous at x = -1.

- [b] Find the value of b so that f(x) is continuous at x = 2.
- [c] If a = 6 and b = 3, find all discontinuities of f(x) and find the type of each discontinuity (removable, jump or infinite).

[10] Use the Intermediate Value Theorem to prove that the equation $\cos 2x = x^2$ has a solution in the interval $[0, \pi]$.

[11] Find all horizontal and vertical asymptotes of
$$f(x) = \frac{\sqrt{4+9x^2}}{2x-1}$$
.

- [12] If $f(x) = x^3 3x + 2$, find f'(-2) using both definitions of f'(a).
- [13] Find a function f and a number a such that the derivative of f at a is given by

[a]
$$\lim_{h \to 0} \frac{\cos(\pi(h-1)) + 1}{h}$$
 [b] $\lim_{x \to -2} \frac{x^2 - x - 6}{x + 2}$

[14] The position of an object (in feet) at time t minutes, is given by the function $f(t) = \sqrt{t^2 - 5}$. Find the instantaneous velocity of the object at time t = 3. Specify the units.

[15] Find the equation of the tangent line to the curve of $f(x) = \frac{2x}{1-x}$ at x = 2.

[16] The graph of f is shown to the right. Arrange the following from least (most negative) to greatest (most positive).



[17] The time required to defrost a piece of frozen meat in the refrigerator depends on the temperature inside the refrigerator. Let

- t = f(T), where t is the defrost time (in hours), and T is the refrigerator temperature (in C)
- [a] Give the practical meaning (including units) of f(4) = 6.
- [b] Give the practical meaning (including units) of f'(4) = -1.
- [c] Is there a value of T_0 for which you would expect $f'(T_0) > 0$? Why or why not?
- [18] Using the definition of the derivative, find the derivatives of the following functions.

[a]
$$f(t) = \frac{1}{\sqrt{1-t}}$$
 [b] $g(x) = \frac{4x}{2-x}$

- [19] The graph of f(x) is shown on the right.
 - [a] Find all x-coordinates where f'(x) is undefined, and explain briefly why.
 - [b] Sketch a graph of f'(x).



[20] If the tangent line to the graph of y = f(x) at x = 4 is x - 2y = 6, prove that $\lim_{x \to 4} f(x) = -1$.

YOU MUST ALSO KNOW THE FOLLOWING DEFINITIONS AND THEOREMS:

Definitions

vertical/horizontal asymptote continuity at a point removable discontinuity (from lecture) jump discontinuity (from lecture) derivative at a point derivative function

Theorems

Squeeze Theorem Intermediate Value Theorem Differentiability implies continuity